**Beam Physics Note 79**
06 August, 2004

# Luminosity Evolution for Lead Ion Beams in the LHC

Amy Nicholson

**Summary**

Precise predictions for the evolution of beam luminosity in time are necessary to determine performance limitations for ion beams in the LHC. We have developed a notebook in Mathematica which implements many of the physics processes to be considered, and have performed calculations for the different cases.

## Introduction

The evolution of luminosity in time is of interest for all particle accelerators in the consideration of operation time versus fill time. Calculations for the luminosity of lead ion beams in the LHC differ from those for proton beams and from those for other lead ion colliders, leading to a new set of operational limitations. In particular, the LHC will be the first heavy-ion collider in which synchrotron radiation will have significant effects on emittance damping due to the high energy of the ion beams.

Luminosity is defined as the interaction rate per unit cross section

$$\mathcal{L} = \frac{f_0\, k_b\, N_{b1}(t)\, N_{b2}(t)}{\pi \left(\sqrt{\beta_{x1}\epsilon_{x1}(t)} + \sqrt{\beta_{x2}\epsilon_{x2}(t)}\right)\left(\sqrt{\beta_{y1}\epsilon_{y1}(t)} + \sqrt{\beta_{y2}\epsilon_{y2}(t)}\right)}$$

where $k_b$ is the number of bunches, $f_0$ is the frequency of revolution, $N_{b1}(t)$ and $N_{b2}(t)$ are the numbers of ions per bunch in each beam, $\beta_{x1}$, $\beta_{x2}$, $\beta_{y1}$, and $\beta_{y2}$ are the x- and y-beta functions for each beam, respectively, and $\varepsilon_{x1}(t)$, $\varepsilon_{x2}(t)$, $\varepsilon_{y1}(t)$, and $\varepsilon_{y2}(t)$ are the x- and y-emittances for each beam, respectively. The variables $N_{b1}$ and $\varepsilon$ evolve with time and are the basis for these calculations.

Estimates of the luminosity evolution are made by solving the set of coupled differential equations for the emittances and intensities of the beams

$$N_b'(t) = \frac{-N_b(t)}{\tau_{bg}} - \sigma_{tot}\mathcal{L}$$

$$\epsilon_{x,y}'(t) = \frac{1}{\tau_{ms}} - \frac{2\epsilon_{x,y}(t)}{\tau_x} + \frac{\epsilon_{x,y}(t)}{\tau_{t,ibs}[N_b(t), \epsilon_{x,y}(t), \epsilon_l(t)]}$$

$$\epsilon_l'(t) = -\frac{4\epsilon_l(t)}{\tau_x} + \frac{\epsilon_l(t)}{\tau_{l,ibs}[N_b(t), \epsilon_{x,y}(t), \epsilon_l(t)]} - D_{RF}$$

where $\sigma_{tot}$ is the total cross section for beam collision losses, $\tau_{bg}$ is the intensity lifetime due to interactions with residual gases, $\tau_{ms}$ is the emittance lifetime due to multiple scattering from residual gases, $\tau_x$ is the synchrotron radiation damping time, $\tau_{t,ibs}$ and $\tau_{l,ibs}$ are functions giving the transverse and longitudinal intrabeam growth times, respectively, and $D_{RF}$ represents artificial diffusion due to RF noise. Separate, but identical, equations are used to solve for the second beam.

We have developed a Mathematica notebook to solve these equations and to calculate various derived quantities, such as the luminosity. The notebook has the ability to simplify the equations if degeneracies exist, such as equal intensities in both beams, as well as the ability to turn physics processes on and off in order to determine their effects on the evolution. The design parameters used to calculate physical effects are given in [1] and [2].

# Physical effects

## *Beam-Beam Interactions*

The dominant causes of particle loss during collision are photo-nuclear interactions, namely Electron Capture by Pair Production (ECPP) and Electromagnetic Dissociation (ED). The total cross section for particle loss, $\sigma_{tot}$, includes the hadronic, ECPP, and ED cross sections, and has a value of 513.756 barns for Pb ions, compared with 0.105 barns for protons.

## *Intra-beam Scattering (IBS)*

(This section by J.M. Jowett)

The IBS growth times in the differential equations could be simply evaluated by defining functions that perform the growth rate calculations using any of the numerous programs or analytical approximation that are available. For example, it is easy to use the Madtomma package to construct functions that return results from the implementation of the Bjorken-Mtingwa theory in MAD. However each evaluation involves a numerical integration at every element in the ring and this direct method is rather slow for the LHC.

A more practical and equally accurate alternative is to use a separate Mathematica notebook to make a scan of the transverse and longitudinal emittance space at constant bunch intensity for given beam conditions (optics, energy). This takes about an hour of CPU time on an lxplus node. The results are condensed into a Mathematica **InterpolatingFunction** object. Knowing that growth rates are directly proportional to intensity, function definitions for the growth times, as functions of intensity and the emittances, are then be built and loaded into the time-evolution notebook. A given set of definitions is valid for the given optics (e.g. Pb ions at LHC Collision energy). Numerical integration of the differential equations is then very fast.

The initial value for the transverse growth time is approximately 13 hours, while the longitudinal growth time is about 7.7 hours.

## *Beam-Vacuum interactions*

Particle loss dominates beam-gas interactions for ions. The intensity lifetime due to beam-gas collisions is given by the following

$$\tau_{bg}^{-1} = \left(\sum_g n_g \sigma_{bg}\right) v_b$$

where $v_b$ is the beam velocity. The total cross sections for the lead beam interacting with residual gases are calculated using abrasion-ablation [3] (hadronic cross-section) and RELDIS [4] (ED cross-section). The Mathematica notebook currently has two sets of values for residual gas densities [5]. One takes the values to be simply ¼ of the start-up vacuum conditions for proton beams. The second is less pessimistic and probably more accurate. Using the pessimistic gas densities, the lifetime is about 80 hours, while with the conditioned gas densities the lifetime grows to about 540 hours. This shows a great need for precise predictions of gas densities in each section of the accelerator.

The growth due to multiple scattering is linear in time. The growth time is given by

$$\tau_{ms}^{-1} = \frac{4\pi r_p^2 c Z_{ion}^2}{2\beta^2 \gamma A_{ion}^2} \beta_{ave} \sum_g n_g Z_g (Z_g + 1) \ln\left(183 Z_g^{-\frac{1}{3}}\right) \qquad [6]$$

where

$$\beta_{ave} = \frac{C}{2\pi Q} \simeq 80\,\text{m}$$

is the average beta function throughout the accelerator. Here Q is the tune of the accelerator. The lifetime is approximately 1 x $10^{14}$ hours. Thus it seems that multiple scattering has little effect on luminosity evolution, however, more accurate predictions of residual gas densities in the accelerator could prove otherwise.

### *Synchrotron Radiation*

The value for the betatron damping time, τx/2, is found using the following formula:

$$\tau_x \to \frac{3 c^6 m_{ion}^4 \rho}{2 f_0 m_p \pi (c^4 m_{ion}^2 + c^2 p_{ion}^2)^{3/2} Q_{ion}^2 r_p}$$

The value for ions, 12.6 hours, is about half that for protons and 0.3% of the value for gold ion beams at RHIC.

### *RF Noise*

Radiation damping causes the longitudinal emittance to shrink twice as quickly as it does for the transverse emittance. This causes an increase in beam size due to increased IBS. In order to control beam growth and, consequently, luminosity decay, artificial noise may be added to cause a spread in the longitudinal direction, effectively keeping the longitudinal emittance a constant.

## Implementation

The differential equations are written in their most general form, with separate equations for each of the two beams and for the radial and vertical emittances. Technically, we work with a set of time-dependent functions and a set containing the ordinary differential equations and initial conditions. Rule-based programming is used to make simplications on these sets, i.e. to reduce the number of equations by half if the two beams have identical intensities. Thus the number of time-dependent functions and the number of differential equations and initial conditions is variable but there is no need to keep track of such details.

The differential equations can be written both symbolically and numerically. The numerical form is obtained by applying another set of rules. The differential equations in numerical form, leaving the number of experiments as a free parameter and showing only collision “burn-off”, IBS, and radiation damping are

$$\epsilon_{l1}'[t] == \epsilon_{l1}[t]\left(-0.159 + \frac{5.14\times10^{-5} N_{b1}[t]}{\text{InterpolatingFunction}[\{\{5.\times10^{-11}, 9.5\times10^{-10}\}, \{0.25, 4.25\}\}, \text{<>}][\epsilon_{x1}[t], \epsilon_{l1}[t]]}\right)$$

$$\epsilon_{x1}'[t] == \epsilon_{x1}[t]\left(-0.0792 + \frac{5.14\times10^{-5} N_{b1}[t]}{\text{InterpolatingFunction}[\{\{5.\times10^{-11}, 9.5\times10^{-10}\}, \{0.25, 4.25\}\}, \text{<>}][\epsilon_{x1}[t], \epsilon_{l1}[t]]}\right)$$

$$N_{b1}'[t] == -\frac{3.31\times10^{-19} n_{exp} N_{b1}[t]^2}{\epsilon_{x1}[t]}$$

Note that the IBS term is given as an interpolating function of the emittances. For all numerical calculations, the luminosity is given in $cm^{-2}s^{-1}$, the longitudinal emittance in eVs, and all other quantities in SI. This set of equations, analytical or numerical, and the symbols for the time-dependent functions equations plus initial conditions are precisely the first arguments of the Mathematica functions **DSolve** and **NDSolve** which return the analytical and numerical solutions.

Various initial conditions, vacuum conditions, and ion parameters can be specified, allowing for extensions to protons, injection parameters, beta-tuning, etc. The number of experiments is generally left as a free parameter. All terms include switches to allow the user to see which physics processes are important for a given situation. The solutions are given as rules and can be either analytical or numerical, with numerical solutions given as interpolating functions. This gives an easy way to compute derived quantities, which are defined in the notebook as a separate set of functions. These include lifetimes, average luminosity, and optimal run times, as well as plotting functions for all quantities. Detailed documentation for the functions is given in the notebook.

Analytical solutions are well-known for the simple case of luminosity burn-off. It is trivial to re-derive these. We have obtained further analytical results for various cases. For example, the solution for the luminosity, considering only intensity burn-off and radiation damping, and assuming an equal number of ions per bunch in the two beams, is

$$\mathcal{L} \to \frac{16\,\beta^* e^{\frac{2t}{\tau_x}} \sqrt{\epsilon_{x10}}\,\sqrt{\epsilon_{y10}}\; f_0\, k_b\, N_{b10}^2\, \pi}{\left(8\,\beta^* \sqrt{\epsilon_{x10}}\,\sqrt{\epsilon_{y10}}\;\pi + \left(-1 + e^{\frac{2t}{\tau_x}}\right) f_0\, N_{b10}\, n_{exp}\, \sigma_{tot}\, \tau_x\right)^2}$$

The corresponding average luminosity is

$$\frac{1}{t_{fill} + t_{run}} \int_0^{t_{run}} \mathcal{L}(t)\, dt = \frac{\left(-1 + e^{\frac{2tRun}{\tau_x}}\right) f_0\, k_b\, N_{b10}^2\, \tau_x}{\left(8\beta^* \sqrt{\epsilon_{x10}}\,\sqrt{\epsilon_{y10}}\;\pi + \left(-1 + e^{\frac{2tRun}{\tau_x}}\right) f_0\, N_{b10}\, n_{exp}\, \sigma_{tot}\, \tau_x\right)(t_{fill} + t_{run})}$$

where $t_{run}$ and $t_{fill}$ are the operation and fill times, respectively. A more complicated example, considering beam-gas collisions, beam-beam collisions, and radiation damping, and assuming equal, round beams, gives a luminosity of

$$\mathcal{L} \to \frac{4\beta^* e^{-\frac{2t}{\tau_x}} \epsilon_{x10} f_0 k_b N_{b10}^2 \pi \left(-2 + \left(\sum_g n_g \sigma_{bg}\right) \tau_x v_{ion}\right)^2}{\left(\left(-1 + e^{\frac{t\left(-2 + \left(\sum_g n_g \sigma_{bg}\right)\tau_x v_{ion}\right)}{\tau_x}}\right) f_0 N_{b10} n_{exp} \sigma_{tot} \tau_x + 4\beta^* e^{\frac{t\left(-2 + \left(\sum_g n_g \sigma_{bg}\right)\tau_x v_{ion}\right)}{\tau_x}} \epsilon_{x10} \pi \left(-2 + \left(\sum_g n_g \sigma_{bg}\right) \tau_x v_{ion}\right)\right)^2}$$

Analytical solutions are not possible when the IBS term is represented as an interpolating function. Hence we shall mostly show numerical solutions below.

# Results for LHC

## *Simplified Models*

As a simple example, we consider first the case of luminosity burn-off only. As Fig. 1 shows, the luminosity decays relatively quickly over the first ten hours if experiments are being performed. For two experiments, the time for the luminosity to decay to 1/e of its initial value is about 7 hours.

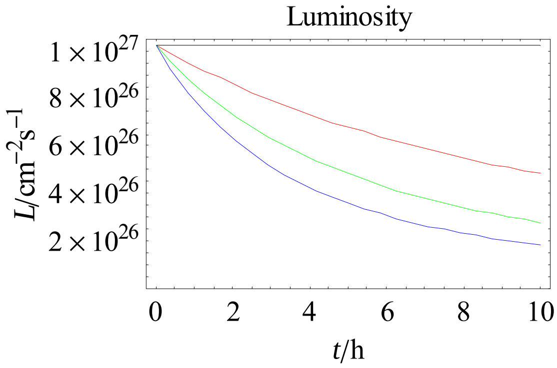



*Fig. 1 Luminosity evolution for Pb ionbeams, assuming burn-off through beam-beam collisions only. The colors (black, red, green, blue) correspond to the number of experiments (0,1,2,3).*

If we now add IBS, the emittance begins to increase with time, causing the luminosity to decay even more quickly. However, adding RF diffusion helps to damp the growth of the emittance, slowing down the luminosity decay (Fig. 2). The RF term increases the luminosity lifetime in this case by about 4%.

Synchrotron radiation greatly increases the luminosity lifetime by damping the growth of the transverse emittance. Fig. 3 shows the effect of adding synchrotron radiation. Rather than initially increasing, the emittance stays relatively constant over a period of 20 hours. This effect is strong enough to compensate for IBS and causes a luminosity decay similar to that for simple burn-off only.

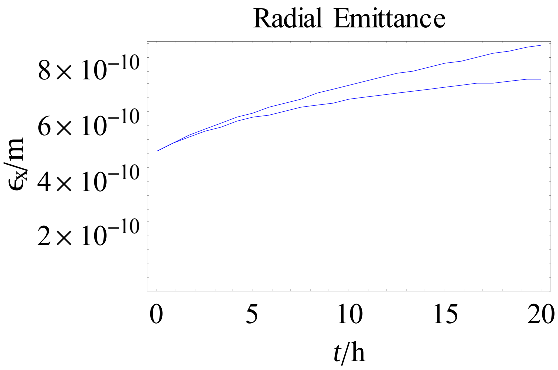



*Fig. 2 Radial emittance, assuming luminosity burn-off and IBS. The lower curve includes RF noise.*

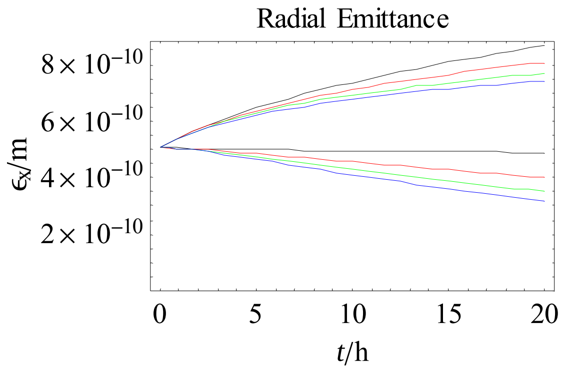
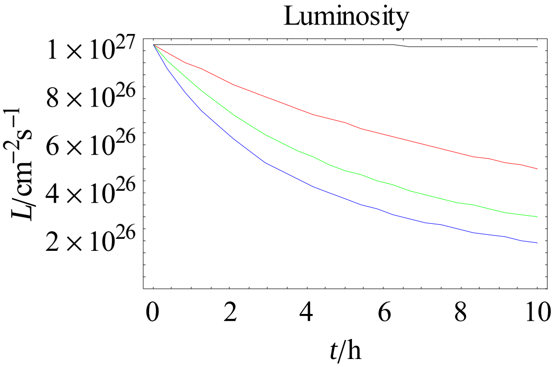



*Fig. 3 Radial emittance. The upper set of curves shows what would happen without synchrotron radiation.*

## *Pb Beams in the LHC*




The most relevant example must include all physics processes. Fig. 4 shows the luminosity evolution for Pb beams in the LHC. The lifetime for two experiments is 7.55 hours. By plotting the average luminosity over 10 hours, assuming a 3 hour fill time, (Fig. 5) we see that for 1 experiment the average luminosity continues to increase, but if more experiments are performed, an optimal run-time is reached within the ten hours (Fig. 6) For 2 experiments, this time is approximately 6 hours.

*Fig. 4 Luminosity evolution for Pb ion beams, assuming all physics.*

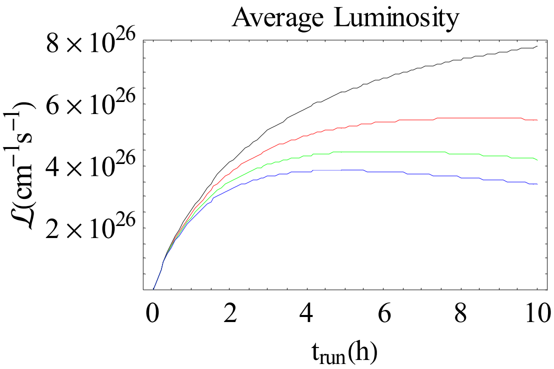



*Fig. 5 Average Luminosity. Assumes a fill time of 3 hours.*

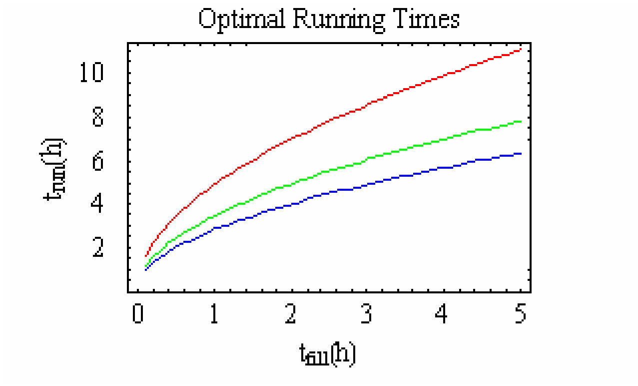
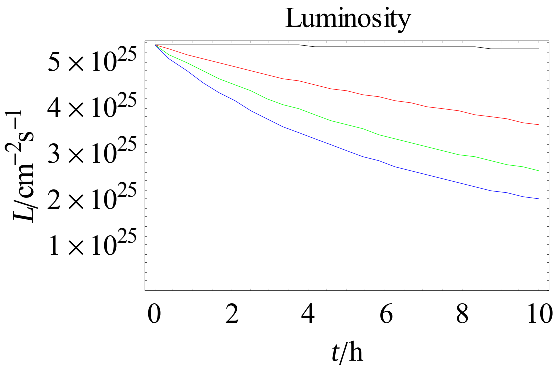



*Fig. 6 Optimal running times vs. fill times.*

## Early Scheme




For the early scheme things look a bit more optimistic, due to an increased beta function and a decreased number of bunches (Fig. _). The lifetime for two experiments is over 15 hours, with an optimal running time of 85 hours for 2 experiments with a 3 hour fill time.

*Fig. 7 Luminosity for Pb ion beams using the parameters for the early scheme.*

## *Different Beams*

In any real accelerator run, differences in beam sizes will occur between the two colliding beams. Fig. 8 shows the intensities of two beams whose intensities and emittances differ by a factor of 0.5. The smaller beam (lower set of curves) decays far more rapidly than the larger beam (upper set of curves), causing the luminosity to decay more quickly than in the case of equal beams (Fig. 9).

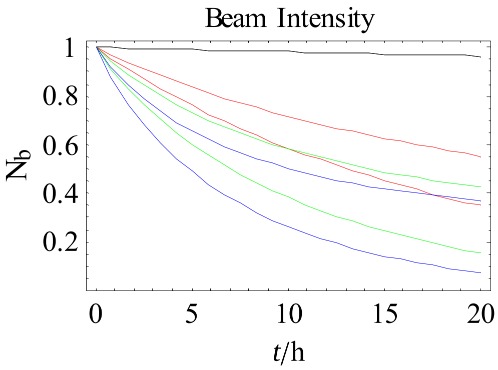



*Fig. 8 Beam Intensities for unequal beams.*

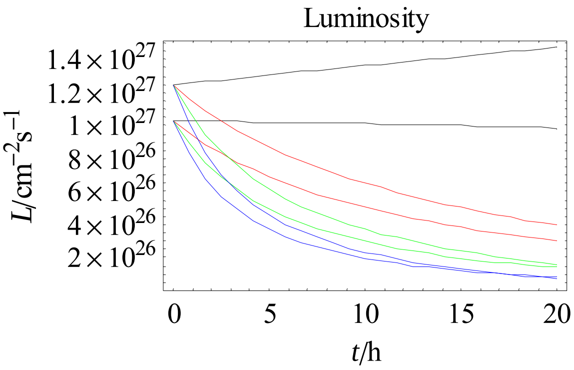



*Fig. 9 Luminosities for unequal beams (upper set of curves) and equal beams (lower set of curves).*

## *Pb Betastar change with time*

The capability for beta-tuning has been implemented into the Mathematica notebook. As an example, we consider the case where the beta function is lowered at four instances during the run-time in an attempt to keep the luminosity approximately constant. The intial value for beta is 1.25 m. It reaches its minimum value of 0.5 m after 7 hours. The initial intensity has been raised to its maximum possible value, about $10^8$ ions per bunch. Fig. 10 shows the resulting luminosity. With this scheme, the average luminosity increases by approximately a factor of 1.3.

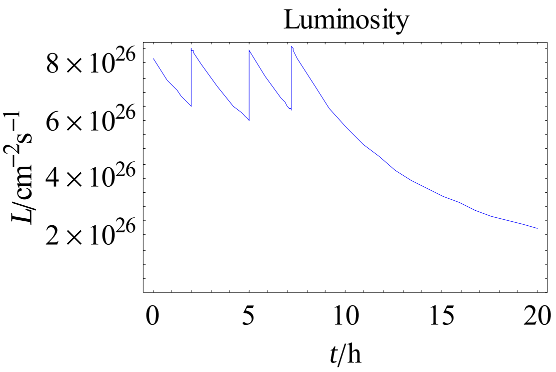



*Fig. 10 Luminosity evolution for 2 experiments, showing the effects of beta-tuning.*

## *Proton examples*

Protons can be studied as a particular case. Considering luminosity burn-off only and using the nominal parameters for proton beams, the luminosity lifetime (time for the luminosity to decrease to 1/e of its initial value) is 22.6 hours for two high luminosity experiments. The average luminosity for 2 experiments with a 3 hour fill time continues to increase for more than 10 hours,.

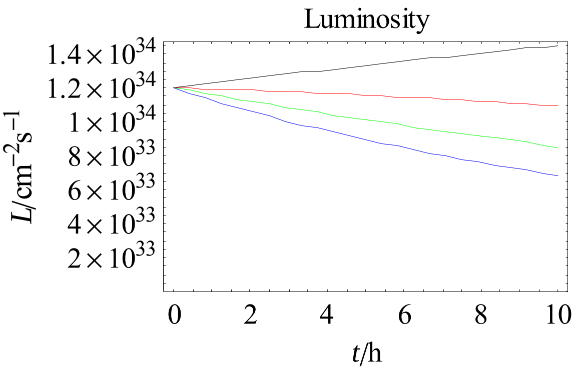



*Fig. 11 Luminosity evolution for proton beams.*

### *Summary/Conclusions*

There are still some improvements to be made to the calculations but we have a flexible framework in which it is easy to plug in improved models of the various physical effects determining the time-evolution of intensity, emittance and luminosity.

More accurate predictions for the gas densities for all sections of the accelerator tube with ion beams still need to be produced, as the requirements for vacuum conditions for ion beams will be more stringent than for protons. In addition, functions for the IBS growth time should be calculated for ion beams at injection and for proton beams.

## References


[1] J.M. Jowett et al, “Heavy Ion Beams in the LHC”, EPAC 2003.

[2] The LHC Design Report, Vol. 1 The LHC Main Ring, CERN-2004-003 Ch. 21.

[3] J.J. Gaimard et al, Nucl. Phys. A531 (1991) 709.

[4] I.A. Pshenichnov et al, Phys. Rev. C64 (2001) 024903.

[5] A. Rossi et al, LHC Project Report 674 (2003).

[6] A. Chao, M. Tigner, *Handbook of Accelerator Physics and Engineering, 2nd Edition*, Ch. 3